FINAL JEE–MAIN EXAMINATION – JANUARY, 2020

(Held On Tuesday 07th JANUARY, 2020) TIME : 2 : 30 PM to 5 : 30 PM

PHYSICS

TEST PAPER WITH ANSWER & SOLUTION

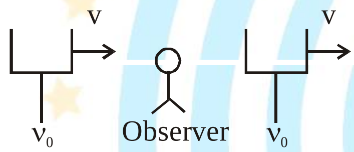
1. A stationary observer receives sound from two identical tuning forks, one of which approaches and the other one recedes with the same speed (much less than the speed of sound). The observer hears 2 beats/sec. The oscillation frequency of each tuning fork is ν_0 = 1400 Hz and the velocity of sound in air is 350 m/s. The speed of each tuning fork is close to :

(1) $\frac{1}{8}$ m/s (2) $\frac{1}{2}$ m/s

(3) 1 m/s (4) $\frac{1}{4}$ m/s

NTA Ans. (4)

Sol.



$\nu_1 = \left(\frac{c}{c-v}\right)\nu_0$

$\nu_2 = \left(\frac{c}{c+v}\right)\nu_0$

beat frequency = $\nu_1 - \nu_2$

$= c\nu_0\left(\frac{1}{c-v} - \frac{1}{c+v}\right)$

$= c\nu_0\left(\frac{c+v-c+v}{c^2-v^2}\right) = \frac{2c\nu_0^2 v}{c^2-v^2}$

$\approx \frac{2c\nu_0 v}{c^2} = \frac{2\nu_0 v}{c} = 2$

$\Rightarrow \frac{2\times1400\times v}{350} = 2$

$\Rightarrow v = \frac{1}{4}$ m/s

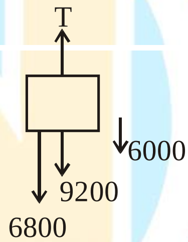
2. An elevator in a building can carry a maximum of 10 persons, with the average mass of each person being 68 kg. The mass of the elevator itself is 920 kg and it moves with a constant speed 3 m/s. The frictional force opposing the motion is 6000 N. If the elevator is moving up with its full capacity, the power delivered by the motor to the elevator (g = 10 m/s²) must be at least :

(1) 56300 W (2) 48000 W

(3) 66000 W (4) 62360 W

NTA Ans. (3)

Sol.



elevater moving with constant speed hence

T = 6800 + 9200 + 6000

T = 22000 N

power = T·v = 22000 × 3

= 66000 W

3. The activity of a radioactive sample falls from 700 s^{-1} to 500 s^{-1} in 30 minutes. Its hafl life is close to :

(1) 66 min (2) 52 min

(3) 72 min (4) 62 min

NTA Ans. (4)

Sol. $A = A_0\left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$

$$500 = 700\left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

$$0.7 \approx \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

$$\left(\frac{1}{2}\right)^{1/2} \approx \frac{t}{T_{1/2}}$$

$$\frac{30}{T_{1/2}} \approx \frac{1}{2} \Rightarrow T_{1/2} = 60$$

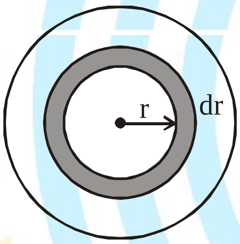
4. Mass per unit area of a circular disc of radius a depends on the distance r from its centre as $\sigma(r) = A + Br$. The moment of inertia of the disc about the the axis, perpendicular to the plane and passing through its centre is :

(1) $2\pi a^4\left(\frac{A}{4}+\frac{aB}{5}\right)$ (2) $\pi a^4\left(\frac{A}{4}+\frac{aB}{5}\right)$

(3) $2\pi a^4\left(\frac{aA}{4}+\frac{B}{5}\right)$ (4) $2\pi a^4\left(\frac{A}{4}+\frac{B}{5}\right)$

NTA Ans. (1)

Sol.



$dI = dmr^2$

$dI = \sigma 2\pi r\, dr\, r^2$

$dI = 2\pi(A + Br)\, r^3\, dr$

$$\int dI = 2\pi\int_0^a \left(Ar^3 + Br^4\right) dr$$

$$I = 2\pi a^4\left(\frac{A}{4}+\frac{B9}{5}\right)$$

5. The electric field of a plane electromagnetic wave is given by $\vec{E} = E_0\frac{\hat{i}+\hat{j}}{\sqrt{2}}\cos(kz+\omega t)$

At t = 0, a positively charged particle is at the point $(x, y, z) = \left(0, 0, \frac{\pi}{k}\right)$. If its instantaneous velocity at (t = 0) is $\upsilon_0\,\hat{k}$, the force acting on it due to the wave is :

(1) zero (2) parallel to $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

(3) antiparallel to $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$ (4) parallel to $\hat{k}$

NTA Ans. (3)

Sol. $\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$

$$\vec{E} = E_0\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)\cos\pi$$

$$= -E_0\frac{\hat{i}+\hat{j}}{\sqrt{2}}$$

as $\vec{E}\times\vec{B} = \vec{c}$

$$+E_0\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)\times\vec{B} = c\hat{k}$$

$$\Rightarrow \vec{B} = -\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)\frac{E_0}{c}$$

$$\vec{F} = q\left(-E_0\frac{(\hat{i}+\hat{j})}{\sqrt{2}} - \frac{v_0\hat{k}}{c}\times(\hat{i}-\hat{j})E_0\right)$$

since $\frac{v_0}{c} << 1$

$\Rightarrow$ F is antiparallel to $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

6. A particle of mass m and charge q has an initial velocity $\vec{\upsilon} = \upsilon_0 \hat{j}$. If an electric field $\vec{E} = E_0 \hat{i}$ and magnetic field $\vec{B} = B_0 \hat{i}$ act on the particle, its speed will double after a time:

(1) $\frac{2m\upsilon_0}{qE_0}$ (2) $\frac{3m\upsilon_0}{qE_0}$

(3) $\frac{\sqrt{3}m\upsilon_0}{qE_0}$ (4) $\frac{\sqrt{2}m\upsilon_0}{qE_0}$

NTA Ans. (3)

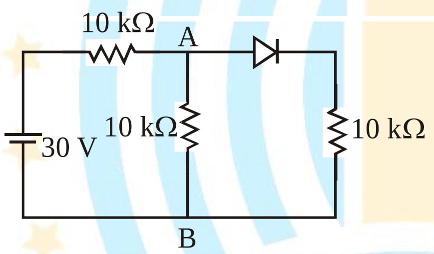
Sol. $(2V_0)^2 = v_0^2 + \upsilon_x^2$

$v_x = \sqrt{3}\, v_0$

$\sqrt{3}\, v_0 = 0 + \frac{qE_0}{m} t$

$t = \frac{\sqrt{3} v_0 m}{qE_0}$

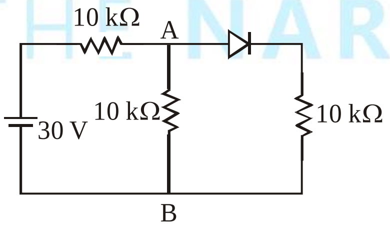
7. In the figure, potential difference between A and B is :

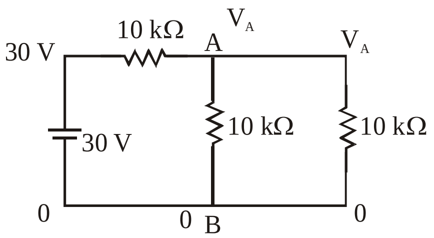


(1) 5V (2) 10 V
(3) zero (4) 15 V

NTA Ans. (2)

Sol.





$$\frac{30 - V_A}{10} + \frac{0 - V_A}{10} + \frac{0 - V_A}{10} = 0$$

$$3 = \frac{3V_A}{10}$$

$V_A = 10$ V

8. The dimension of $\frac{B^2}{2\mu_0}$, where B is magnetic field and μ_0 is the magnetic permeability of vacuum, is:

(1) $ML^{-1}T^{-2}$ (2) ML^2T^{-1}
(3) MLT^{-2} (4) ML^2T^{-2}

NTA Ans. (1)

Sol. Magnetic energy stored per unit volume is

$\frac{B^2}{2\mu_0}$

Dimension is $ML^{-1}T^{-2}$

9. In a building there are 15 bulbs of 45 W, 15 bulbs of 100 W, 15 small fans of 10 W and 2 heaters of 1 kW. The voltage of electric main is 220 V. The minimum fuse capacity (rated value) of the building will be:

(1) 10 A (2) 25 A
(3) 15 A (4) 20 A

NTA Ans. (4)

Sol. $220\,I = P = 15 \times 45 + 15 \times 100 + 15 \times 10 + 2 \times 10^3$

$I = \frac{4325}{220} = 19.66$

$I \simeq 20$ A

10. An emf of 20 V is applied at time t=0 to a circuit containing in series 10 mH inductor and 5 Ω resistor. The ratio of the currents at time $t = \infty$ and at t = 40 s is close to : (Take $e^2 = 7.389$)

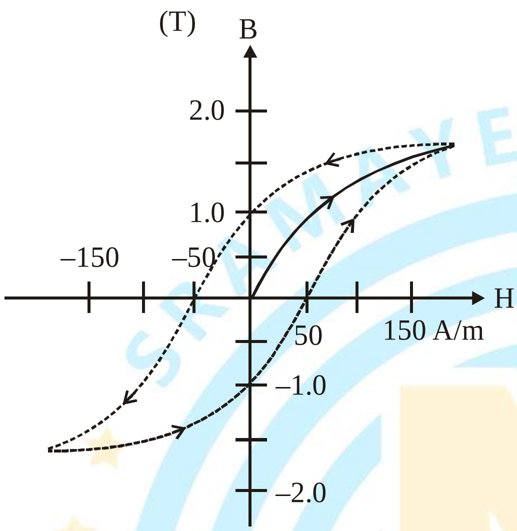
(1) 1.06 (2) 1.15
(3) 1.46 (4) 0.84

NTA Ans. (2)

Sol. $i = i_0 (1 - e^{-Rt/L})$

$\frac{i_0}{i} = \frac{1}{1-e^{-2\times10^4}}$

$\frac{i_0}{i} \simeq 1$

11. 

The figure gives experimentally measured B vs. H variation in a ferromagnetic material. The retentivity, co-ercivity and saturation, respectively, of the material are:

(1) 150 A/m, 1.0 T and 1.5 T

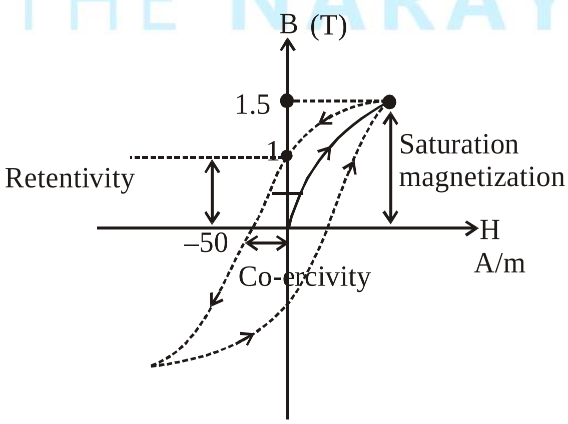
(2) 1.0 T, 50 A/m and 1.5 T

(3) 1.5 T, 50 A/m and 1.0 T

(4) 1.5 T, 50 A/m and 1.0 T

NTA Ans. (2)

Sol.



Retentivity = 1.0 T

Co-ercivity = 50 A/m

Saturation = 1.5 T

12. In a Young's double slit experiment, the separation between the slits is 0.15 mm. In the experiment, a source of light of wavelength 589 nm is used and the interference pattern is observed on a screen kept 1.5 m away. The separation between the successive bright fringes on the screen is:

(1) 6.9 mm (2) 5.9 mm

(3) 4.9 mm (4) 3.9 mm

NTA Ans. (2)

Sol. Finge width, $\beta = \frac{D\lambda}{d} = \frac{1.5\times589\times10^{-9}}{0.15\times10^{-3}}$

$= 5.9 \times 10^{-3}$ m

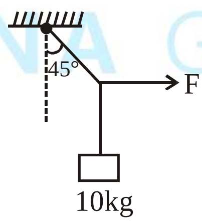
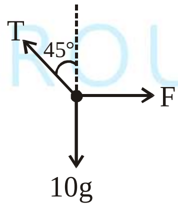
$= 5.9$ mm

13. A mass of 10 kg is suspended by a rope of length 4 m, from the ceiling. A force F is applied horizontally at the mid-point of the rope such that the top half of the rope makes an angle of 45° with the vertical. Then F equals: (Take g= 10 ms^{-2} and the rope to be massless)

(1) 100 N (2) 90 N

(3) 75 N (4) 70 N

NTA Ans. (1)

Sol.  

For equilibrium,

T sin 45° = F ....(1)

and T cos 45° = 10g ....(2)

equation (1)/(2)

we get F = 10g

= 100 N

14. A thin lens made of glass (refractive index = 1.5) of focal length f = 16 cm is immersed in a liquid of refractive index 1.42. If its focal length in liquid is f_1, then the ratio f_l/f is closest to the integer :

(1) 1 (2) 5 (3) 9 (4) 17

NTA Ans. (3)

Sol. Using $\frac{1}{f}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

$$\frac{1}{f}=\left(\frac{1.5}{1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad ....(1)$$

and $\frac{1}{f_1}=\left(\frac{1.5}{1.42}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ ....(2)

equation (1)/(2),

we get $\frac{f_1}{f}=\frac{0.5}{0.056}$

= 8.93 ≈ 9

15. A planar loop of wire rotates in a uniform magnetic field. Initially, at t = 0, the plane of the loop is perpendicular to the magnetic field. If it rotates with a period of 10 s about an axis in its plane then the magnitude of induced emf will be maximum and minimum, respectively at :

(1) 2.5 s and 7.5 s (2) 5.0 s and 7.5s

(3) 5.0 s and 10.0 s (4) 2.5s and 5.0 s

NTA Ans. (4)

Sol. Flux ϕ = $\vec{B}\cdot\vec{A}$ = BA cos θ = BA cos ωt

|Induced emf| = |e| = $\left|\frac{d\phi}{dt}\right|$ = |BAω sin ωt|

|e| will be maximum at ωt = $\frac{\pi}{2}$

$$\left(\frac{2\pi}{T}\right)t=\frac{\pi}{2}$$

$$\left(\frac{2\pi}{10}\right)t=\frac{\pi}{2}\Rightarrow t = 2.5 \text{ sec}$$

|e| will be minimum at ωt = π

$$\left(\frac{2\pi}{10}\right)t=\pi \Rightarrow t = 5 \text{ sec}$$

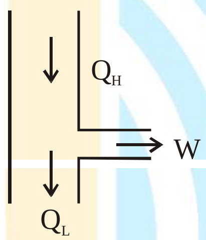
16. Two ideal Carnot engines operate in cascade (all heat given up by one engine is used by the other engine to produce work) between temperatures, T_1 and T_2. The temperature of the hot reservoir of the first engine is T_1 and the temperature of the cold reservoir of the second engine is T_2. T is temperature of the sink of first engine which is also the source for the second engine. How is T related to T_1 and T_2, if both the engines perform equal amount of work ?

(1) $T = \frac{2T_1T_2}{T_1+T_2}$ (2) $T = \sqrt{T_1T_2}$

(3) $T = \frac{T_1+T_2}{2}$ (4) T = 0

NTA Ans. (3)

Sol.



$\frac{Q_H}{Q_L}=\frac{T_1}{T}$ and $W = Q_H - Q_L$ ....(1)

$\frac{Q_L}{Q'_L}=\frac{T}{T_2}$ and $W = Q_L - Q'_L$ ....(2)

17. A box weighs 196 N on a spring balance at the north pole. Its weight recorded on the same balance if it is shifted to the equator is close to (Take g = 10 ms^{-2} at the north pole and the radius of the earth = 6400 km):

(1) 195.66 N (2) 194.66 N

(3) 194.32 N (4) 195.32 N

NTA Ans. (4)

Sol. $W = 196 - m\omega^2R$

18. Under an adiabatic process, the volume of an ideal gas gets doubled. Consequently the mean collision time between the gas molecule changes from τ_1 to τ_2. If $\frac{C_p}{C_v} = \gamma$ for this gas then a good estimate for $\frac{\tau_2}{\tau_1}$ is given by :

(1) $\left(\frac{1}{2}\right)^{\frac{\gamma+1}{2}}$ (2) 2

(3) $\frac{1}{2}$ (4) $\left(\frac{1}{2}\right)^{\gamma}$

NTA Ans. (1)

Sol. $t \propto \frac{V}{\sqrt{T}}$ ....(1)

$TV^{\gamma-1}$ = constant ....(2)

$\therefore\ t \propto V^{\frac{\gamma+1}{2}}$

19. An ideal fluid flows (laminar flow) through a pipe of non-uniform diameter. The maximum and minimum diameters of the pipes are 6.4 cm and 4.8 cm, respectively. The ratio of the minimum and the maximum velocities of fluid in this pipe is:

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{3}{4}$

(3) $\frac{81}{256}$ (4) $\frac{9}{16}$

NTA Ans. (4)

Sol. $A_1 v_1 = A_2 v_2$

$$\frac{v_{min}}{v_{max}} = \frac{A_{min}}{A_{max}}$$

$$\frac{v_{min}}{v_{max}} = \left(\frac{4.8}{6.4}\right)^2$$

$$\frac{v_{min}}{v_{max}} = \frac{9}{16}$$

20. An electron (of mass m) and a photon have the same energy E in the range of a few eV. The ratio of the de-Broglie wavelength associated with the electron and the wavelength of the photon is (c = speed of light in vacuum)

(1) $\left(\frac{E}{2m}\right)^{1/2}$ (2) $\frac{1}{c}\left(\frac{E}{2m}\right)^{1/2}$

(3) $c(2mE)^{1/2}$ (4) $\frac{1}{c}\left(\frac{2E}{m}\right)^{1/2}$

NTA Ans. (2)

20. $\frac{\lambda_{electron}}{\lambda_{photon}} = ?$

$E = \frac{hc}{\lambda_{photon}}$ ....(1)

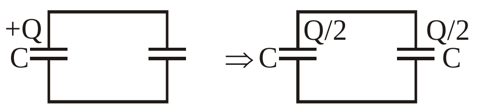
$\lambda_{electron} = \frac{h}{\sqrt{2mE}}$ ....(2)

from (1) and (2)

$$\frac{\lambda_{electron}}{\lambda_{photon}} = \frac{1}{c}\left(\frac{E}{2m}\right)^{1/2}$$

21. A 60 pF capacitor is fully charged by a 20 V supply. It is then disconnected from the supply and is connected to another uncharged 60 pF capactior is parallel. The electrostatic energy that is lost in this process by the time the charge is redistributed between them is (in nJ) ____.

NTA Ans. (6)

Sol. 

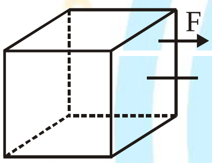
$Q = CV$

$$\Delta Q_L = \frac{Q^2}{2C} - \left[\frac{(Q/2)^2}{2C} \times 2\right] = \frac{Q^2}{4C}$$

$$= \frac{1}{4} CV^2$$

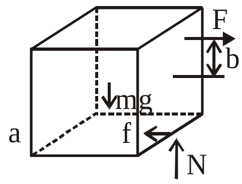
$$= \frac{1}{4} \times 60 \times 10^{-12} \times 4 \times 10^2$$

$= 6nJ$

22. 

Consider a uniform cubical box of side a on a rough floor that is to be moved by applying minimum possible force F at a point b above its centre of mass (see figure). If the coefficient of friction is $\mu = 0.4$, the maximum possible value of $100 \times \frac{b}{a}$ for a box not to topple before moving is ______ .

NTA Ans. (75)

Sol. 

$F = \mu mg$ ...(1)

$F\left(b + \frac{a}{2}\right) = mg \frac{a}{2}$ ...(2)

$$\mu mg \left(b + \frac{a}{2}\right) = mg \times \frac{a}{2}$$

$$\left(b + \frac{a}{2}\right) \mu = \frac{a}{2}$$

$$0.4 = \mu = \frac{a}{2b + a}$$

$0.8b + 0.4a = a$

$0.8\ b = 0.6\ a$

$$\frac{b}{a} = \frac{3}{4}$$

23. The balancing length for a cell is 560 cm in a potentiometer experiment. When an external resistance of 10 Ω is connected in parallel to the cell, the balancing length changes by 60cm. If the internal resistance of the cell is $\frac{N}{10}\Omega$, where N is an integer then value of N is____ .

NTA Ans. (12)

Sol. $r = R\left(\frac{x - x'}{x'}\right)$

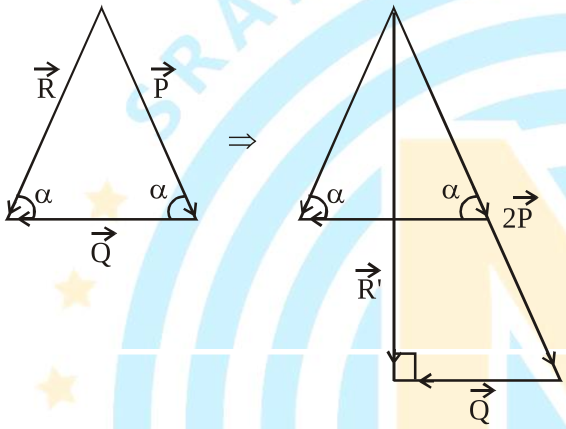
$$= 10 \times \frac{60}{500}$$

$= 12$

24. The sum of two forces $\vec{P}$ and $\vec{Q}$ is $\vec{R}$ such that $|\vec{R}| = |\vec{P}|$. The angle θ (in degrees) that the resultant of $2\vec{P}$ and $\vec{Q}$ will make with $\vec{Q}$ is, _____ .

NTA Ans. (90)

Sol.



Hence angle 90°

25. M grams of steam at 100°C is mixed with 200 g of ice at its melting point in a thermally insulated container. If it produces liquid water at 40°C [heat of vaporization of water is 540 cal/ g and heat of fusion of ice is 80 cal/g], the value of M is______.

NTA Ans. (40)

Sol. $M \times 540 + M + 60 = 200 \times 80 + 200 \times 1 \times (40 - 0)$

$\Rightarrow M = 40$

FINAL JEE–MAIN EXAMINATION – JANUARY, 2020

(Held On Tuesday 07th JANUARY, 2020) TIME : 2 : 30 PM to 5 : 30 PM

CHEMISTRY

TEST PAPER WITH ANSWER & SOLUTION

1. Within each pair of elements of F & Cl , S & Se, and Li & Na, respectively, the elements that release more energy upon an electron gain are-

(1) F, Se and Na

(2) F, S and Li

(3) Cl, S and Li

(4) Cl, Se and Na

NTA Ans. (3)

Sol. (i) Electron affinity of second period p-block element is less than third period p-block element due to small size of second period p-block element.

E.A. order : F < Cl

(ii) Down the group electron affinity decreases due to size increases.

EA. order : S > Se

Li > Na

2. The redox reaction among the following is :

(1) Combination of dinitrogen with dioxygen at 2000 K

(2) Formation of ozone from atmosphereic oxygen in the presence of sunlight

(3) Reaction of H_2SO_4 with NaOH

(4) Reaction of $[Co(H_2O)_6]Cl_3$ with $AgNO_3$

NTA Ans. (1)

Sol. (i) $N_2 + O_2 \xrightarrow{2000\ K} 2NO$ (Redox reaction)

during the reaction, oxidation of nitrogen take place from 0 to 2 and reduction of oxygen take place from 0 to –2. It means this reaction is redox reaction.

(ii) $3O_2 \xrightarrow{h\nu} 2O_3$ (Non – redox reaction)

(iii) $H_2SO_4 + 2NaOH \rightarrow Na_2SO_4 + 2H_2O$ (neutralization reaction)

(iv) $[Co(H_2O)_6]Cl_3 + 3AgNO_3 \rightarrow 3AgCl\downarrow + [Co(H_2O)_6](NO_3)_3$ (White ppt.)

3. Among the statements(a)-(d), the incorrect ones are-

(a) Octahedral Co(III) complexes with strong field ligands have very high magnetic moments

(b) When $\Delta_0 < P$, the d-electron configuration of Co(III) in an octahedral complex is $t_{eg}^4\ e_g^2$

(c) Wavelength of light absorbed by $[Co(en)_3]^{3+}$ is lower than that of $[CoF_6]^{3-}$

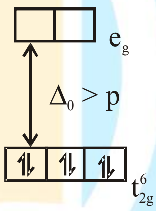
(d) If the Δ_0 for an octahedral complex of Co(III) is 18,000 cm^{-1}, the Δ_t for its tetrahedral complex with the same ligand will be 16,000 cm^{-1}

(1) (a) and (b) only (2) (c) and (d) only

(3) (b) and (c) only (4) (a) and (d) only

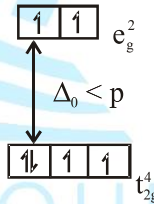
NTA Ans. (4)

Sol. (a) Co^{+3} (with strong field ligands)



diamagnetic

(b) If $\Delta_0 < p$;



(c) Splitting power of ethylenediamine (en) is greater than fluoride (F^-) ligand therefore more energy absorbed by $[Co(en)_3]^{3+}$ as compared to $[CoF_6]^{3-}$.

So wave length of light absorbed by $[Co(en)_3]^{3+}$ is lower than that of $[CoF_6]^{3-}$

(d) $\Delta_t = \frac{4}{9}\Delta_0$

so if Δ_0 = 18,000 cm^{-1}

$\Delta_t = \frac{4}{9} \times 18000 = 8000\ cm^{-1}$

Statement (a) and (d) are incorrect.

4. The number of possible optical isomers for the complexes MA_2B_2 with sp^3 and dsp^2 hybridised metal atom, respectively, is :

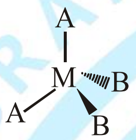
Note : A and B are unidentate neutral and unidentate monoanionic ligands, respectively

(1) 0 and 0 (2) 0 and 2

(3) 0 and 1 (4) 2 and 2

NTA Ans. (1)

Sol. (a) If the complex MA_2B_2 is sp^3 hybridised then the shape of this complex is tetrahedral this structure is opticaly inactive due to the presence of plane of symmetry.



Optical isomes = 0

(b) If the complex MA_2B_2 is dsp^2 hybridised then the shape of this complex is square planar.

(Cis-isomer) (Trans-isomer)

Both isomers are optically inactive due to the presence of plane of symmetry.

Optical isomers = 0

5. In the following reactions products(A) and (B), respectively , are :

$NaOH + Cl_2 \rightarrow (A) + \text{side products}$

(hot and conc.)

$Ca(OH)_2 + Cl_2 \rightarrow (B) + \text{side products}$

(dry)

(1) $NaClO_3$ and $Ca(OCl)_2$

(2) $NaOCl$ and $Ca(ClO_3)_2$

(3) $NaClO_3$ and $Ca(ClO_3)_2$

(4) $NaOCl$ and $Ca(OCl)_2$

NTA Ans. (1)

Sol. $6NaOH + 3Cl_2 \longrightarrow NaClO_3 + 5NaCl + 3H_2O$

(hot and conc.) (A) side product

$2Ca(OH)_2 + 2Cl_2 \longrightarrow Ca(OCl)_2 + CaCl_2 + 2H_2O$

dry (B) side product

6. Which of the following statements is correct-

(1) Gluconic acid can form cyclic (acetal/hemiacetal) structure

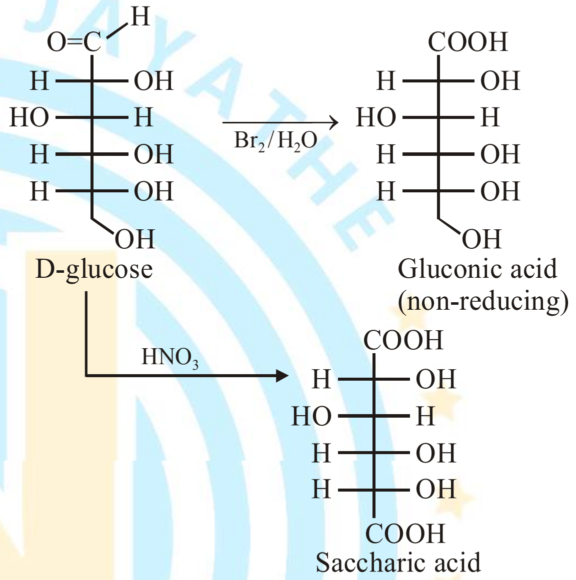
(2) Gluconic acid is a partial oxidation product of glucose

(3) Gluconic acid is obtained by oxidation of glucose with HNO_3

(4) Gluconic acid is a dicarboxylic acid

NTA Ans. (2)

Sol.



7. The bond order and the magnetic characteristics of CN^- are :

(1) 3, diamagnetic

(2) $2\frac{1}{2}$, paramagnetic

(3) 3, paramagnetic

(4) $2\frac{1}{2}$, diamagnetic

NTA Ans. (1)

Sol. According to MOT (If z is internuclear axis)

The configuration of

$$CN^- : \sigma_{1s}^2, \sigma_{1s}^{*2}, \sigma_{2s}^2, \sigma_{2s}^{*2}, \pi_{2p_x}^2 = \pi_{2p_y}^2, \sigma_{2p_z}^2$$

$$\text{Bond order} = \frac{1}{2}(10-4)$$

$$= 3$$

CN^- is diamagnetic due to absence of unpaired electron

8. The equation that is incorrect is -

(1) $\left(\Lambda_m^0\right)_{NaBr} - \left(\Lambda_m^0\right)_{NaI} = \left(\Lambda_m^0\right)_{KBr} - \left(\Lambda_m^0\right)_{NaBr}$

(2) $\left(\Lambda_m^0\right)_{H_2O} = \left(\Lambda_m^0\right)_{HCl} + \left(\Lambda_m^0\right)_{NaOH} - \left(\Lambda_m^0\right)_{NaCl}$

(3) $\left(\Lambda_m^0\right)_{KCl} - \left(\Lambda_m^0\right)_{NaCl} = \left(\Lambda_m^0\right)_{KBr} - \left(\Lambda_m^0\right)_{NaBr}$

(4) $\left(\Lambda_m^0\right)_{NaBr} - \left(\Lambda_m^0\right)_{NaCl} = \left(\Lambda_m^0\right)_{KBr} - \left(\Lambda_m^0\right)_{KCl}$

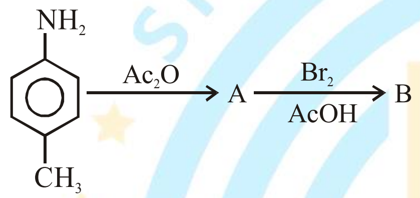
NTA Ans. (1)

Sol. Option (1) is incorrect.
According to Kohlrausch's law correct expression is

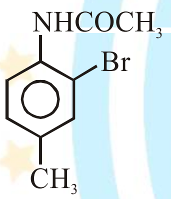
$\left(\Lambda_m^0\right)_{NaBr} - \left(\Lambda_m^0\right)_{NaI} = \left(\Lambda_m^0\right)_{KBr} - \left(\Lambda_m^0\right)_{KI}$

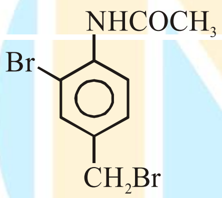
The other statements are correct.

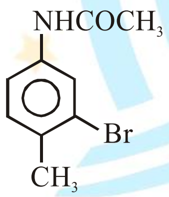
9. In the following reaction sequence

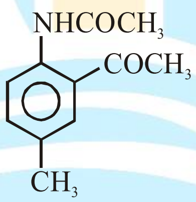


the major products B is -

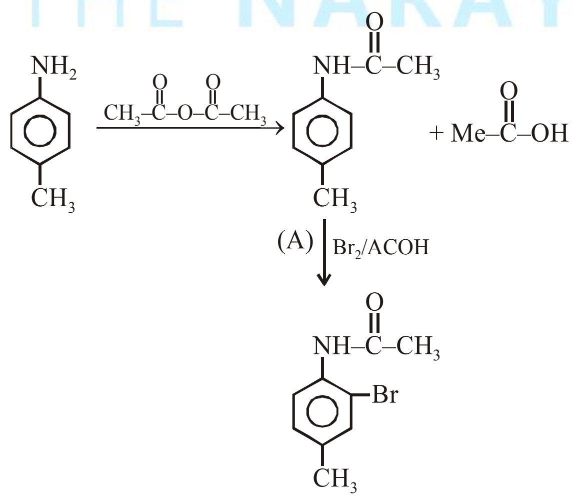
(1)


(2)


(3)


(4)


NTA Ans. (1)

Sol.


10. Two open beakers one containing a solvent and the other containing a mixture of that solvent with a non volatile solute are together sealed in a container. Over time -

(1) The volume of the solution does not change and the volume of the solvent decreases

(2) The volume of the solution decrease and the volume of the solvent increases

(3) The volume of the solution increase and the volume of the solvent decreases

(4) The volume of the solution and the solvent does not change

NTA Ans. (3)

Sol. The pure solvent solution will try to maintain higher vapour pressure in the sealed container and in return the solvent vapour molecules will condense in the solution of non-volatile solute as it maintains an equilibrium with lower vapour pressure. (Lowering of vapour pressure is observed when a non volatile solute is mixed in a volatile solvent)

This will eventually lead to increase in the volume of solution and decrease in the volume of solvent.

11. A chromatography column, packed with silica gel as stationary phase, was used to separate a mixture of compounds consisting of (A) benzanilide (B) aniline and (C) acetophenone. When the column is eluted with a mixture of solvents, hexane : ethyl acetate (20 : 80), the sequence of obtained compounds :

(1) (B), (C) and (A) (2) (C), (A) and (B)

(3) (A), (B) and (C) (4) (B), (A) and (C)

NTA Ans. (2)

Sol. (A) Benzanilide → Ph–NH–C(=O)–Ph (μ = 2.71 D)

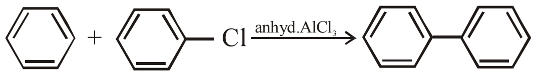
(B) Aniline → Ph–NH_2 (μ = 1.59 D)

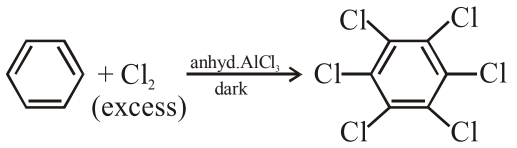
(C) Acetophenone → Ph–C(=O)–CH_3 (μ = 3.05 D)

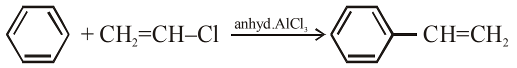
Dipole moment : C > A > B

Hence the sequence of obtained compounds is (C), (A) and (B)

12. Consider the following reactions :

(a) 

(b) 

(c) 

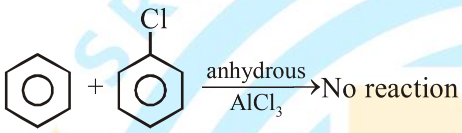
(d) 

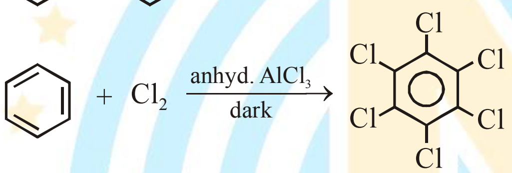
Which of these reactions are possible ?

(1) (a) and (d) (2) (b) and (d)

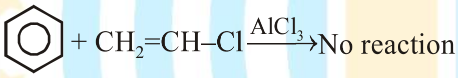
(3) (a) and (b) (4) (b) , (c) and (d)

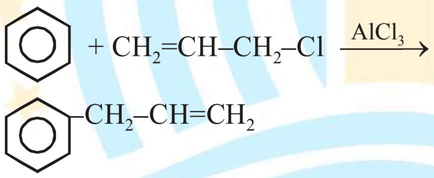
NTA Ans. (2)

Sol. (a) 

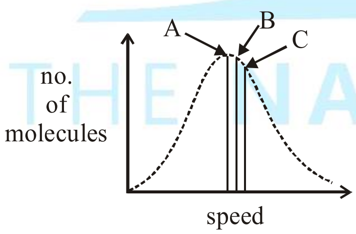
(b) 

(electrophilic substitution)

(c) 

(d) 

13. Identify the correct labels of A, B and C in the following graph from the options given below:



Root mean square speed (V_{rms}) ; most probable speed (V_{mp}) ; Average speed ($V_{av.}$)

(1) A – V_{rms} ; B –V_{mp} ; C – V_{av}

(2) A – V_{av} ; B –V_{rms} ; C – V_{mp}

(3) A – V_{mp} ; B –V_{rms} ; C – V_{av}

(4) A – V_{mp} ; B –V_{av} ; C – V_{rms}

NTA Ans. (4)

Sol. $V_{mp}\left(=\sqrt{\frac{2RT}{M}}\right) < V_{av}\left(=\sqrt{\frac{8RT}{\pi M}}\right) < V_{rms}\left(=\sqrt{\frac{3RT}{M}}\right)$

14. Among the statements (a) - (d), the correct ones are -

(a) Decomposition of hydrogen peroxide gives dioxygen

(b) Like hydrogen peroxide, compounds , such as $KClO_3$, $Pb(NO_3)_2$ and $NaNO_3$ when heated liberated dioxygen

(c) 2-Ethylanthraquinone is useful for the industrial preparation of hydrogen peroxide.

(d) Hydrogen peroxide is used for the manufacture of sodium perborate

(1) (a), (b) and (c) only

(2) (a) and (c) only

(3) (a), (b) , (c) and (d)

(4) (a), (c) and (d) only

NTA Ans. (3)

Sol. (a) $H_2O_2 \rightarrow 2H_2O + O_2$

(b) $KClO_3 \xrightarrow[MnO_2]{\Delta} KCl + \frac{3}{2}O_2$

$Pb(NO_3)_2 \xrightarrow{\Delta} PbO + 2NO_2 + \frac{1}{2}O_2$

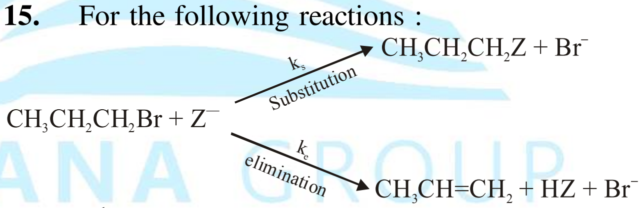
$NaNO_3 \xrightarrow{\Delta} NaNO_2 + \frac{1}{2}O_2$

(c) 2-ethylanthraquinol $\underset{H_2/Pd}{\overset{O_2(air)}{\rightleftharpoons}}$ 2-ethylanthraquinone + H_2O_2

(d) $2H_3BO_3 + 2NaOH + 2H_2O_2 \longrightarrow Na_2[B_2(O_2)(OH)_4] + 4H_2O$

All statements are correct

15. For the following reactions :



where

$Z^- = CH_3CH_2O^-$ (A) or $H_3C-\underset{CH_3}{\overset{CH_3}{C}}-O^-$ (B),

k_s and k_e, are , respectively, the rate constants for the substitution and elimination, and $\mu = \frac{k_s}{k_e}$, the correct options is -

(1) $\mu_B > \mu_A$ and $k_e(B) > k_e(A)$

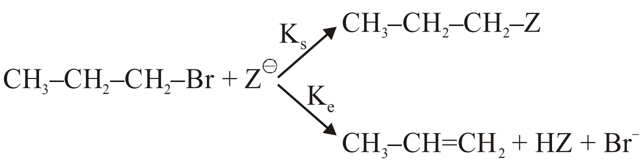
(2) $\mu_B > \mu_A$ and $k_e(A) > k_e(B)$

(3) $\mu_A > \mu_B$ and $k_e(B) > k_e(A)$

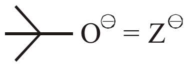
(4) $\mu_A > \mu_B$ and $k_e(A) > k_e(B)$

NTA Ans. (3)

Sol.



(A) $CH_3–CH_2–O^- = Z^{\ominus} \longrightarrow$

(B) 

(B) with more steric crowding forms elimination product compared to substitution.

$K_e(B) > K_e(A)$

$$\mu_B = \frac{K_s(B)}{K_e(A)} < \mu_A = \frac{K_s(A)}{K_e(A)}$$

16. The refining method used when the metal and the impurities have low and high melting temperatures, respectively, is -

(1) zone refining

(2) liquation

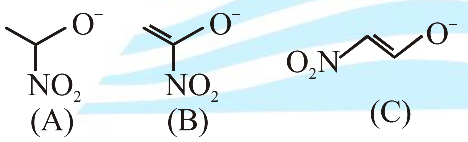
(3) vapour phase refining

(4) distillation

NTA Ans. (2)

Sol. Liquation method is used when the melting point of metal is less compare to the melting point of the associated impurity.

17. The correct order of stability for the following alkoxides is :



(1) (C) > (B) > (A)

(2) (C) > (A) > (B)

(3) (B) > (C) > (A)

(4) (B) > (A) > (C)

NTA Ans. (1)

Sol. (C) > (B) > (A)

18. The ammonia (NH_3) released on quantitative reaction of 0.6 g urea (NH_2CONH_2) with sodium hydroxide (NaOH) can be neutralized by :

(1) 100 ml of 0.1 N HCl

(2) 200 ml of 0.4 N HCl

(3) 100 ml of 0.2 N HCl

(4) 200 ml of 0.2 N HCl

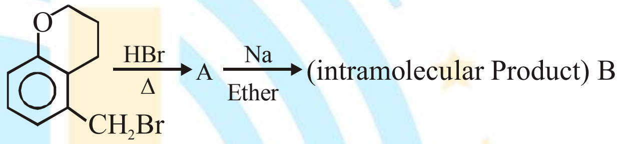
NTA Ans. (3)

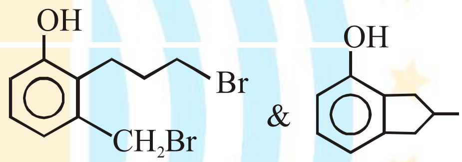
Sol. $NH_2CONH_2 + 2NaOH \rightarrow Na_2CO_3 + 2NH_3$

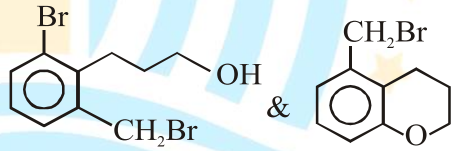
10 mmoles   20mmoles

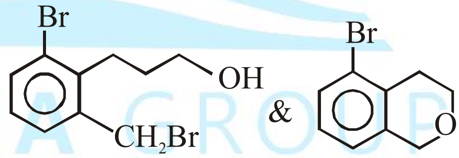
Hence, NH_3 will require 20 meq.

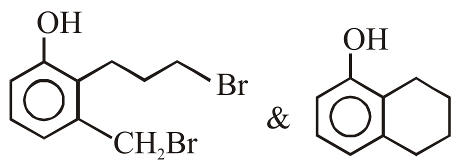
19. In the following reaction squence, structures of A and B, respectively will be :



(1) 

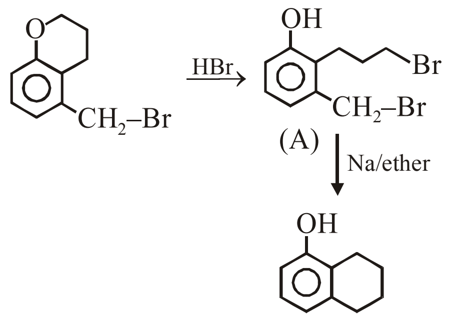
(2) 

(3) 

(4) 

NTA Ans. (4)

Sol.



20. For the reaction

$2H_2(g) + 2NO(g) \rightarrow N_2(g) + 2H_2O(g)$

the observed rate expression is, rate = $k_f[NO]^2[H_2]$. The rate expression of the reverse reaction is :

(1) $k_b[N_2][H_2O]^2/[NO]$ (2) $k_b[N_2][H_2O]$

(3) $k_b[N_2][H_2O]^2$ (4) $k_b[N_2][H_2O]^2/[H_2]$

NTA Ans. (4)

Sol. $K_{eq} = \frac{k_f}{k_b} = \frac{[N_2][H_2O]^2}{[H_2]^2[NO]^2}$

At equilibrium $r_f = r_b$

$$k_f \underset{[Given]}{[H_2][NO]^2} = k_b \frac{[N_2][H_2O]^2}{[H_2]}$$

Hence, rate expression for reverse reaction.

$$= k_b \frac{[N_2][H_2O]^2}{[H_2]}$$

21. Conside the following reactions :

$NaCl + K_2Cr_2O_7 + H_2SO_4(Conc.) \rightarrow (A) +$ Side products

$(A) + NaOH \rightarrow (B) +$ Side product

$(B) + H_2SO_4(dilute) + H_2O_2 \rightarrow (C) +$ Side product

The sum of the total number of atoms in one molecule each of (A), (B) and (C) is

NTA Ans. (18.00 to 18.00)

Sol. $4NaCl + K_2Cr_2O_7 + 6H_2SO_4$
$\downarrow$
$\underset{(A)}{2CrO_2Cl_2} + 4NaHSO_4 + 2KHSO_4 + 3H_2O$

$CrO_2Cl_2 + 4\,NaOH \longrightarrow \underset{(B)}{Na_2CrO_4} + 2NaCl + 2H_2O$

$Na_2CrO_4 + 2H_2SO_4 + 2H_2O_2$
$\downarrow$
$\underset{(C)}{CrO_5} + 2NaHSO_4 + 3H_2O$

$A = CrO_2Cl_2$

$B = Na_2CrO_4$

$C = CrO_5$

Total number of atom in A + B + C = 18

22. 3g of acetic acid is added to 250 mL of 0.1 M HCl and the solution made up to 500 mL.

To 20 mL of this solution $\frac{1}{2}$ mL of 5 M NaOH is added. The pH of the solution is _____.

[Given : pK_a of acetic acid = 4.75, molar mass of acetic acid = 60 g/mol, log3 = 0.4771]

Neglect any changes in volume

NTA Ans. (5.22 to 5.24)

Sol. 3gm Acetic Acid + 250 ml 0.1 M HCl +Water $\longrightarrow$ made to 500 ml solution.

$\Rightarrow$ 500 ml solution has 25 meq of HCl
50 meq of CH_3COOH

$\therefore$ 20ml solution has 1 meq of HCl
2 meq of CH_3COOH

We have added 2.5 meq. of NaOH $\left(5M, \frac{1}{2} ml\right)$

Finally , NaOH & HCl are completely consumed and we are left with 0.5 meq of CH_3COOH and 1.5 meq of CH_3COONa

$$pH = pKa + \log \frac{1.5}{0.5}$$
$$= 4.75 + \log 3 = 4.75 + 0.4771$$
$$= 5.2271$$

23. The standard heat of formation $\left(\Delta_f H^0_{298}\right)$ of ethane in (kJ/mol), if the heat of combustion of ethane, hydrogen and graphite are –1560, –393.5 and –286 kJ/mol, respectively is _______

NTA Ans. (–192.00 to –193.00)

Sol. $2C(\text{graphite}) + 3H_2(g) \longrightarrow C_2H_6(g)$

$\Delta_f H\ (C_2H_6) = 2\Delta H_{comb}\ (C_{graphite}) + 3\ \Delta H_{comb}(H_2)$
$- \Delta H_{comb}\ (C_2H_6)$

$= -(286\times 2) - (393.5\times 3) - (-1560)$

$= -572 - 1180.5 + 1560 = -192.5$ kJ/mole

24. The flucculation value of HCl for arsenic sulphide sol. is 30 m mole L^{-1}. If H_2SO_4 is used for the flocculation of arsenic sulphide, the amount, in grams, of H_2SO_4 in 250 ml required for the above purpose is ______.

(molecular mass of H_2SO_4 = 98 g/mol)

NTA Ans. (0.36 to 0.38)

Sol. 1 L solution requires 30 m.mol HCl
250 ml sol. will require 7.5 m.mol HCl
or 3.75 m.mol H_2SO_4

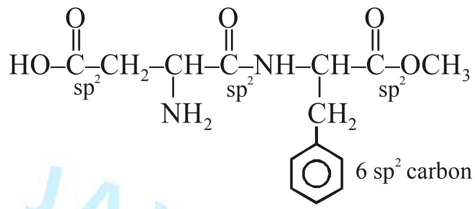
$\Rightarrow \frac{3.75\times 98}{1000}$ gm H_2SO_4

$= 0.3675$ gm H_2SO_4

25. The number of sp^2 hybridised carbons present in "Aspartame" is ________.

NTA Ans. (9.00 to 9.00)

Sol.



no. of sp^2–carbon → 9

FINAL JEE–MAIN EXAMINATION – JANUARY, 2020

(Held On Tuesday 07th JANUARY, 2020) TIME : 2 : 30 PM to 5 : 30 PM

MATHEMATICS

TEST PAPER WITH ANSWER & SOLUTION

1. Let y = y(x) be a function of x satisfying $y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$ where k is a constant and $y\left(\frac{1}{2}\right) = -\frac{1}{4}$. Then $\frac{dy}{dx}$ at $x = \frac{1}{2}$, is equal to:

(1) $\frac{\sqrt{5}}{2}$ (2) $-\frac{\sqrt{5}}{2}$

(3) $\frac{2}{\sqrt{5}}$ (4) $-\frac{\sqrt{5}}{4}$

NTA Ans. (2)

Sol. Put x = sinθ, y = sinα

$y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$

$\Rightarrow$ sinα · cosθ + cosα · sinθ = k

$\Rightarrow$ sin(α + θ) = k

$\Rightarrow$ $\alpha + \theta = \sin^{-1}k$

$\Rightarrow$ $\sin^{-1}x + \sin^{-1}y = \sin^{-1}k$

$$\Rightarrow \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-y^2}} \times \frac{dy}{dx} = 0$$

at $x = \frac{1}{2}$, $y = \frac{-1}{4}$

$$\frac{dy}{dx} = \frac{-\sqrt{5}}{2}$$

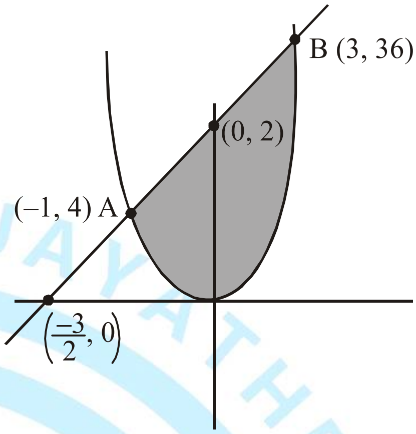
2. The area (in sq. units) of the region $\{(x, y) \in R^2 | 4x^2 \le y \le 8x + 12)$ is :

(1) $\frac{127}{3}$ (2) $\frac{125}{3}$

(3) $\frac{124}{3}$ (4) $\frac{128}{3}$

NTA Ans. (4)

Sol. $4x^2 - y \le 0$ and $8x - y + 12 \ge 0$



On solving y = 4x²

and y = 8x + 12

We get A (–1, 4) & B(3, 36)

Required area = area of the shaded region

$$= \int_{-1}^{3} (8x + 12 - 4x^2)dx = \frac{128}{3}$$

3. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three units vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. If $\lambda = \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ and $\vec{d}=\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$, then the ordered pair, $(\lambda, \vec{d})$ is equal to :

(1) $\left(-\frac{3}{2}, 3\vec{a}\times\vec{b}\right)$ (2) $\left(-\frac{3}{2}, 3\vec{c}\times\vec{b}\right)$

(3) $\left(\frac{3}{2}, 3\vec{b}\times\vec{c}\right)$ (4) $\left(\frac{3}{2}, 3\vec{a}\times\vec{c}\right)$

NTA Ans. (1)

Sol. $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}\cdot\vec{b})+2(\vec{b}\cdot\vec{c})+2(\vec{c}\cdot\vec{a})=0$

$\lambda = \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a} = \frac{-3}{2}$

$\vec{d} = \vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$

$\vec{a}+\vec{b}+\vec{c}=\vec{0}$

$\Rightarrow \vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

$\Rightarrow \vec{d} = 3(\vec{a}\times\vec{b})$

4. If the sum of the first 40 terms of the series, 3 + 4 + 8 + 9 + 13 + 14 + 18 +19 +... is (102)m, then m is equal to :

(1) 20 (2) 5 (3) 10 (4) 25

NTA Ans. (1)

Sol. Sum of the 40 terms of

3 + 4 + 8 + 9 + 13 + 14 + 18 + 19...

= (3 + 8 + 13 +...upto 20 term)

+ [4 + 9 + 15 +... upto 20 terms]

= 10 [{6 + 19 × 5} + {8 + 19 × 5}]

= 10 × 204 = 20 × 102

5. The value of c in the Lagrange's mean value theorem for the function $f(x) = x^3 - 4x^2 + 8x + 11$, when $x \in [0, 1]$ is :

(1) $\frac{2}{3}$ (2) $\frac{\sqrt{7}-2}{3}$

(3) $\frac{4-\sqrt{5}}{3}$ (4) $\frac{4-\sqrt{7}}{3}$

NTA Ans. (4)

Sol. $f(0) = 11$

$f(1) = 16$

$$\frac{f(1)-f(0)}{1-0} = 3c^2 - 8c + 8$$

$\Rightarrow 3c^2 - 8c + 8 = 5$

$\Rightarrow 3c^2 - 8c + 3 = 0$

$c \in [0, 1] \Rightarrow c = \frac{4-\sqrt{7}}{3}$

6. If θ_1 and θ_2 be respectively the smallest and the largest values of θ in $(0, 2\pi) - \{\pi\}$ which satisfy the equation, $2\cot^2\theta - \frac{5}{\sin\theta} + 4 = 0$, then $\int_{\theta_1}^{\theta_2} \cos^2 3\theta d\theta$ is equal to :

(1) $\frac{2\pi}{3}$ (2) $\frac{\pi}{3} + \frac{1}{6}$ (3) $\frac{\pi}{9}$ (4) $\frac{\pi}{3}$

NTA Ans. (4)

Sol. $2\cos^2\theta - 5\sin\theta + 4\sin^2\theta = 0$

$3\sin^2\theta - 5\sin\theta + 2 = 0$

$\sin\theta = \frac{1}{2}, 2$ (Rejected)

$$\int_{\theta_1}^{\theta_2} \cos^2 3\theta d\theta = \int_{\pi/6}^{5\pi/6} \frac{1+\cos 6\theta}{2} d\theta$$

$$= \frac{1}{2}\left(\frac{5\pi}{6} - \frac{\pi}{6}\right) = \frac{2\pi}{6} = \frac{\pi}{3}$$

7. The number of ordered pairs (r, k) for which $6 \cdot {}^{35}C_r = (k^2 - 3) \cdot {}^{36}C_{r+1}$, where k is an integer, is :

(1) 3 (2) 2 (3) 4 (4) 6

NTA Ans. (3)

Sol. $6 \times {}^{35}C_r = (k^2 - 3)\,{}^{36}C_{r+1}$

$k^2 - 3 > 0 \Rightarrow k^2 > 3$

$$k^2 - 3 = \frac{6 \times {}^{35}C_r}{{}^{36}C_{r+1}} = \frac{r+1}{6}$$

Possible values of r for integral values of k, are

r = 5, 35

number of ordered pairs are 4

(5, 2), (5, –2), (35, 3), (35, –3)

8. Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two 3 × 3 real matrices such that $b_{ij} = (3)^{(i+j-2)} a_{ji}$, where i, j = 1, 2, 3. If the determinant of B is 81, then the determinant of A is :

(1) 3 (2) 1/3 (3) 1/81 (4) 1/9

NTA Ans. (4)

Sol. $b_{ij} = (3)^{(i+j-2)} a_{ij}$

$$B = \begin{bmatrix} a_{11} & 3a_{12} & 3^2 a_{13} \\ 3a_{21} & 3a_{22} & 3a_{23} \\ 3^2 a_{31} & 3^2 a_{32} & 3^2 a_{33} \end{bmatrix}$$

$$\Rightarrow |B| = 3 \times 3^2 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 3a_{21} & 3a_{22} & 3a_{23} \\ 3^2 a_{31} & 3^2 a_{32} & 3^2 a_{33} \end{vmatrix}$$

$= 3^6|A|$

$$\Rightarrow |A| = \frac{81}{27 \times 27} = \frac{1}{9}$$

9. Let $a_1, a_2, a_3, \ldots$ be a G.P. such that $a_1 < 0$, $a_1 + a_2 = 4$ and $a_3 + a_4 = 16$. If $\sum_{i=1}^{9} a_i = 4\lambda$, then λ is equal to :

(1) –171 (2) 171 (3) $\frac{511}{3}$ (4) –513

NTA Ans. (1)

Sol. $a_1 + a_2 = 4$

$r^2a_1 + r^2a_2 = 16$

$\Rightarrow r^2 = 4 \Rightarrow r = -2$ as $a_1 < 0$

and $a_1 + a_2 = 4$

$a_1 + a_1(-2) = 4 \Rightarrow a_1 = -4$

$$4\lambda = (-4)\left(\frac{(-2)^9 - 1}{-2-1}\right) = (-4) \times \frac{513}{3}$$

$\Rightarrow \lambda = -171$

10. Let A, B, C and D be four non-empty sets. The contrapositive statement of "If $A \subseteq B$ and $B \subseteq D$, then $A \subseteq C$" is :

(1) If $A \subseteq C$, then $B \subset A$ or $D \subset B$

(2) If $A \not\subseteq C$, then $A \not\subseteq B$ or $B \not\subseteq D$

(3) If $A \not\subseteq C$, then $A \subseteq B$ and $B \subseteq D$

(4) If $A \not\subseteq C$, then $A \not\subseteq B$ and $B \subseteq D$

NTA Ans. (2)

Sol. Contrapositive of $p \to q$ is $\sim q \to \sim p$

$(A \subseteq B) \wedge (B \subseteq D) \longrightarrow (A \subseteq C)$

Contrapositive is

$\sim(A \subseteq C) \longrightarrow \sim(A \subseteq B) \vee \sim(B \subseteq D)$

$A \not\subseteq C \to (A \not\subseteq B) \vee (B \not\subseteq D)$

11. If $3x + 4y = 12\sqrt{2}$ is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$ for some $a \in R$, then the distance between the foci of the ellipse is :

(1) 4 (2) $2\sqrt{7}$ (3) $2\sqrt{5}$ (4) $2\sqrt{2}$

NTA Ans. (2)

Sol. $3x + 4y = 12\sqrt{12}$ is tangent to $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$

$c^2 = m^2a^2 + b^2$

$\Rightarrow a^2 = 16$

$$e = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}$$

Distance between focii = 2ae = $2\sqrt{7}$

12. The value of α for which $4\alpha\int_{-1}^{2} e^{-\alpha|x|}dx = 5$, is :

(1) $\log_e\left(\frac{3}{2}\right)$ (2) $\log_e\left(\frac{4}{3}\right)$

(3) $\log_e 2$ (4) $\log_e \sqrt{2}$

NTA Ans. (3)

Sol. $$4\alpha\left[\int_{-1}^{0} e^{\alpha x}dx + \int_{0}^{2} e^{-\alpha x}dx\right] = 5$$

$$\Rightarrow 4\alpha\left(\left[\frac{e^{\alpha x}}{\alpha}\right]_{-1}^{0} + \left[\frac{e^{-\alpha x}}{-\alpha}\right]_{0}^{2}\right) = 5$$

$\Rightarrow 4e^{-2\alpha} + 4e^{-\alpha} - 3 = 0$

Let $e^{-\alpha} = t$, $4t^2 + 4t - 3 = 0$, $t = \frac{1}{2}, \frac{-3}{2}$ (Rejected)

$e^{-\alpha} = \frac{1}{2}$; $\alpha = \ell n2$

13. The coefficient of x^7 in the expression $(1 + x)^{10} + x(1 + x)^9 + x^2(1 + x)^8 + \ldots + x^{10}$ is :

(1) 120 (2) 330 (3) 210 (4) 420

NTA Ans. (2)

Sol. Coefficient of x^7 is

$${}^{10}C_7 + {}^{9}C_6 + {}^{8}C_5 + \ldots + {}^{4}C_1 + {}^{3}C_0$$

$$\underbrace{{}^{4}C_0 + {}^{4}C_1}_{{}^{5}C_1} + {}^{5}C_2 + \ldots + {}^{10}C_7 = {}^{11}C_7 = 330$$

14. Let α and β be the roots of the equation $x^2 - x - 1 = 0$. If $p_k = (\alpha)^k + (\beta)^k$, $k \geq 1$, then which one of the following statements is not true ?

(1) $(p_1 + p_2 + p_3 + p_4 + p_5) = 26$

(2) $p_5 = 11$

(3) $p_3 = p_5 - p_4$

(4) $p_5 = p_2 \cdot p_3$

NTA Ans. (4)

Sol. $\alpha + \beta = 1, \alpha\beta = -1$

$P_k = \alpha^k + \beta^k$

$\alpha^2 - \alpha - 1 = 0$

$\Rightarrow \alpha^k - \alpha^{k-1} - \alpha^{k-2} = 0$

$\&\ \beta^k - \beta^{k-1} - \beta^{k-2} = 0$

$\Rightarrow P_k = P_{k-1} + P_{k-2}$

$P_1 = \alpha + \beta = 1$

$P_2 = (\alpha + \beta)^2 - 2\alpha\beta = 1 + 2 = 3$

$P_3 = 4$

$P_4 = 7$

$P_5 = 11$

15. The locus of the mid-points of the perpendiculars drawn from points on the line, $x = 2y$ to the line $x = y$ is :

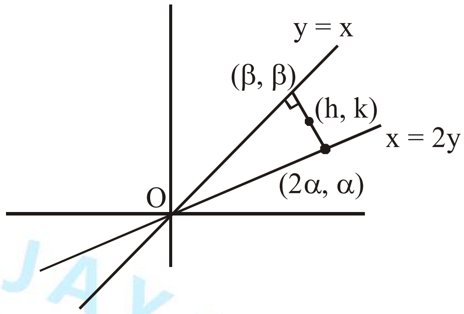
(1) $2x - 3y = 0$ (2) $7x - 5y = 0$

(3) $5x - 7y = 0$ (4) $3x - 2y = 0$

NTA Ans. (3)

Sol. $\dfrac{\alpha - \beta}{2\alpha - \beta} = -1$

$3\alpha = 2\beta$



$h = \dfrac{2\alpha + \beta}{2}$

$2h = \dfrac{7\alpha}{2}$

$k = \dfrac{\alpha + \beta}{2}$

$2k = \dfrac{5\alpha}{2}$

$\dfrac{h}{k} = \dfrac{7}{5}$

$5x = 7y$

16. If $\dfrac{3 + i\sin\theta}{4 - i\cos\theta}, \theta \in [0, 2\pi]$, is a real number, then an argument of $\sin\theta + i\cos\theta$ is :

(1) $-\tan^{-1}\left(\dfrac{3}{4}\right)$ (2) $\tan^{-1}\left(\dfrac{4}{3}\right)$

(3) $\pi - \tan^{-1}\left(\dfrac{4}{3}\right)$ (4) $\pi - \tan^{-1}\left(\dfrac{3}{4}\right)$

NTA Ans. (3)

Sol. $\dfrac{3 + i\sin\theta}{4 - i\cos\theta}$ is a real number

$\Rightarrow 3\cos\theta + 4\sin\theta = 0$

$\Rightarrow \tan\theta = \dfrac{-3}{4}$

argument of $\sin\theta + i\cos\theta = \pi - \tan^{-1}\dfrac{4}{3}$

17. Let y = y(x) be the solution curve of the differential equation, $(y^2 - x)\frac{dy}{dx} = 1$, satisfying y(0) = 1. This curve intersects the x-axis at a point whose abscissa is :

(1) 2 + e (2) 2

(3) 2 – e (4) –e

NTA Ans. (3)

Sol. $(y^2 - x)\frac{dy}{dx} = 1$

$\Rightarrow \frac{dx}{dy} + x = y^2$

I.F. = $e^{\int dy} = e^y$

Solution is given by

$x e^y = \int y^2 e^y dy + C$

$\Rightarrow x e^y = (y^2 - 2y + 2)e^y + C$

x = 0, y = 1, gives C = –e

If y = 0, then x = 2 – e

18. Let f(x) be a polynomial of degree 5 such that x = ±1 are its critical points. If $\lim_{x\to 0}\left(2 + \frac{f(x)}{x^3}\right) = 4$, then which one of the following is not true?

(1) f is an odd function

(2) x = 1 is a point of minima and x = –1 is a point of maxima of f.

(3) x = 1 is a point of maxima and x = –1 is a point of minimum of f.

(4) f(1) – 4f(–1) = 4

NTA Ans. (2)

Sol. $\lim_{x\to 0}\left(2 + \frac{f(x)}{x^3}\right) = 4$

$\Rightarrow f(x) = 2x^3 + ax^4 + bx^5$

$f'(x) = 6x^2 + 4ax^3 + 5bx^4$

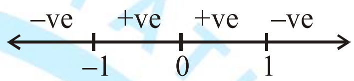
$f'(1) = 0,\ f'(-1) = 0$

$a = 0,\ b = \frac{-6}{5} \Rightarrow f(x) = 2x^3 - \frac{6}{5}x^5$

$f'(x) = 6x^2 - 6x^4$

$= 6x^2(1 - x)(1 + x)$

Sign scheme for f'(x)



Minima at x = –1

Maxima at x = 1

19. In a workshop, there are five machines and the probability of any one of them to be out of service on a day is $\frac{1}{4}$. If the probability that at most two machines will be out of service on the same day is $\left(\frac{3}{4}\right)^3 k$, then k is equal to :

(1) $\frac{17}{2}$ (2) 4

(3) $\frac{17}{8}$ (4) $\frac{17}{4}$

NTA Ans. (3)

Sol. Probability that at most 2 machines are out of service

$= {}^5C_0\left(\frac{3}{4}\right)^5 + {}^5C_1\left(\frac{3}{4}\right)^4\left(\frac{1}{4}\right) + {}^5C_2\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^2$

$= \left(\frac{3}{4}\right)^4 \times \frac{17}{8} \qquad \Rightarrow k = \frac{17}{8}$

20. Let the tangents drawn from the origin to the circle, $x^2 + y^2 - 8x - 4y + 16 = 0$ touch it at the points A and B. The $(AB)^2$ is equal to :

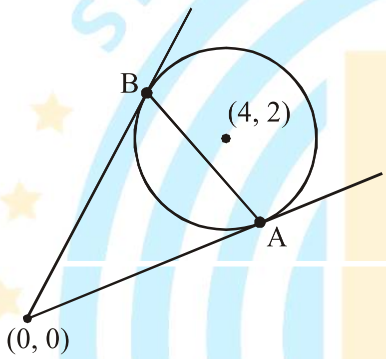
(1) $\frac{52}{5}$ (2) $\frac{32}{5}$

(3) $\frac{56}{5}$ (4) $\frac{64}{5}$

NTA Ans. (4)

Sol. $R = \sqrt{16+4-16} = 2$

$L = \sqrt{S_1} = 4$



AB(Chord of contact) = $\frac{2LR}{\sqrt{L^2+R^2}} = \frac{8}{\sqrt{5}}$

$(AB)^2 = \frac{64}{5}$

21. If the system of linear equations,

$x + y + z = 6$

$x + 2y + 3z = 10$

$3x + 2y + \lambda z = \mu$

has more two solutions, then $\mu - \lambda^2$ is equal to ________

NTA Ans. (13.00)

Sol. System has intfinitely many solution

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 2 & \lambda \end{vmatrix} = 0$$

$\Rightarrow \lambda = 1$

$$D_1 = \begin{vmatrix} 6 & 1 & 1 \\ 10 & 2 & 3 \\ \mu & 2 & 1 \end{vmatrix} = 0$$

$\mu = 14$

$\mu - \lambda^2 = 13$

22. If the function f defined on $\left(-\frac{1}{3}, \frac{1}{3}\right)$ by

$$f(x) = \begin{cases} \frac{1}{x}\log_e\left(\frac{1+3x}{1-2x}\right) & , \text{when } x \neq 0 \\ k & , \text{when } x = 0 \end{cases}$$

is continuous, then k is equal to_______

NTA Ans. (5.00)

Sol. $k = \lim_{x \to 0}\left(\frac{\ell n(1+3x)}{x} - \frac{\ell n(1-2x)}{x}\right)$

$k = 3 + 2 = 5$

23. If the mean and variance of eight numbers 3, 7, 9, 12, 13, 20, x and y be 10 and 25 respectively, then x·y is equal to_____

NTA Ans. (54.00)

Sol. $\frac{3+7+9+12+13+20+x+y}{8}=10$

$x + y = 16$

$\frac{\Sigma x^2}{n}-\left(\frac{\Sigma x}{n}\right)^2=25$

$3^2 + 7^2 + 9^2 + 12^2 + 13^2 + 20^2 + x^2 + y^2 = 1000$

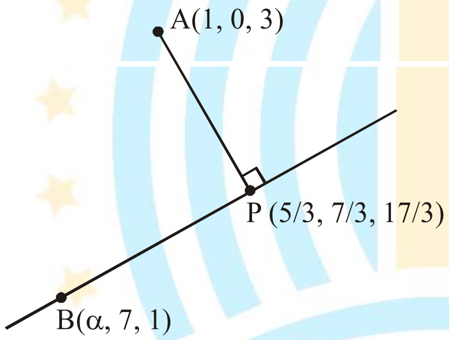
$x^2 + y^2 = 148$

$xy = 54$

24. If the foot of the perpendicular drawn from the point (1, 0, 3) on a line passing through (α, 7, 1) is $\left(\frac{5}{3},\frac{7}{3},\frac{17}{3}\right)$, then α is equal to_____

NTA Ans. (4.00)

Sol.



D.R. of BP = $<\frac{5}{3}-\alpha,\frac{7}{3}-7,\frac{17}{3}-1>$

D.R. of AP = $<\frac{5}{3}-1,\frac{7}{3}-0,\frac{17}{3}-3>$

BP $\perp^r$ AP

$\Rightarrow \alpha = 4$

25. Let X = {n ∈ N : 1 ≤ n ≤ 50}. If

A = {n ∈ X : n is a multiple of 2} and

B = {n ∈ X : n is a multiple of 7}, then the number of elements in the smallest subset of X containing both A and B is_____

NTA Ans. (29.00)

Sol. $n(A) = 25$

$n(B) = 7$

$n(A \cap B) = 3$

$n(A \cup B) = 25 + 7 - 3 = 29$